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Skewness is the Name of the Game

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Abstract

Theoretical models of risk taking attempt to explain why risk-averse individuals participate in unfair gambles. This paper evaluates the two explanations as to why rational individuals would accept gambles with negative expected returns. It is found that it is skewness, not the mean or the variance of the prize distribution that attracts risk-averse gamblers. However, evidence shows that there seems to be an optimal trade-off between operators’ sales revenues and skewness of the pay-off; a point that designer of gambling games needs to heed to.

Key Words: Skewness, mean-variance approach, gambling games, unfair gambles, horseracing, lotteries

JEL Classification No: D81, D84, L83

I. Introduction

An individual must make billions of decisions over a lifetime and a lot of these decisions involve outcomes that are uncertain at the time of decision making. This implies that decision making, no matter big or small, is about evaluating possible outcomes taking into account the likelihood of their realisation. Economists in the past few decades have been using the von Neumann-Morgenstern theory of expected utility to predict individuals' decision making with risk[1]. This theory postulates that individuals will choose the course of actions that maximise their expected utilities. That is, given *m* actions $(a_1, \cdots, a_m)$ and *n* possible outcomes $(x_{j1}, \cdots, x_{jn})$ with corresponding set of probabilities $(r_{j1}, \cdots, r_{jn})$, an individual's choice boils down to choosing an action a_j among the *m* actions such that the highest expected utility or in layman terms anticipated satisfaction is achieved. Mathematically, the problem is:

$$\max_{a_j} E\left(U\left(a_j\right)\right) = \sum_{i=1}^{n} r_{ji}\, U\left(x_{ji}\right) \tag{1}$$

And it is only reasonable to make a decision that brings about a positive or zero expected utility. Yet, what has continued to puzzle economists is that there are many incidences in gambling and investment (both involve the transfer of money among parties based on the outcome of some contingencies) where individuals have chosen actions that yield negative expected monetary returns, therefore a negative expected utility. This seemingly paradoxical behaviour has attracted much attention and effectuated many debates among academia, especially from the disciplines of economics and psychology.

This paper re-examines the validity of the two arguments put forward by various economists to explain the anomaly. It is found that the skewness, not the mean or the variance of the prize distribution, attracts gamblers and investors resulting in the 'irrational' behaviour mentioned above. The reminder of this paper is organised as follows: Section II examines the phenomenon of accepting gambles with negative expected returns. Section III discusses the validity of the two arguments explaining the rationality of accepting gambles with negative expected returns. Section IV examines the findings of recent empirical studies on the anomaly. Section V discusses the implication of the love of skewness on the optimal design of gambles. Section VI is the conclusion.

II. Negative Expected Returns in Horseracing and Lotteries

Researchers have long observed a phenomenon known as long shot bias in horseracing. By long shot bias, we mean that favourites tend to be under bet, win more often than the market odds indicate and have positive expected returns while long shots tend to be over bet, win less often than the market odds indicate and have negative expected returns. Ziemba & Hausch (1986) examine the data of more than 5,000 races in California racecourses and observe a steep drop in expected returns per dollar bet for market odds above 18 to 1 (all the way to 200 to 1) in their studies of US racetracks. According to the long shot bias, horses with market odds above 18 to 1 are long shots and they win less than the market odds indicate. Other studies in straight bets in horseracing (see e.g., Rosett, 1965; Thaler & Ziemba, 1988) also reveal this anomaly.

This phenomenon of race goers being attracted particularly to bets with much longer odds to win—therefore more likely to have negative expected returns—is also confirmed from casual observations in Australia and overseas. In Australian racecourses, there is a shift from straight bets towards quinella, a much riskier bet than the straight bet. In Hong Kong, the introduction of exotic bets (bets involving the outcome of more than one race) such as Double[1], Six Up[2], Double Quinella[3], Treble[4], Double Trio[5], Triple Trio[6] have captured the imagination of race goers. The money placed in straight bets has dwindled to only a tiny percentage of the betting pool. Technically, exotic bets exemplify the long shot bias in straight bets by substantially lowering the market odds[7] of winning the bets. Taking Triple Trio as an example, assume that there are 14 starters in each race (the maximum number of starters allowed on the narrow racecourses of Hong Kong) and that each horse has an equal chance to be in the money. The consequent odds of qualifying for the dividend are about 48.23m to 1. For the 18 race meetings from 18 November 1998 to 27 January 1999, the average Triple Trio dividend is $4.0m per ticket with each ticket costing $2.10. Even though half of the horses in each race have no hope of placing in the top three places, the average return would still be well below the market odds, implying negative expected returns.

This phenomenon of presumably rational individuals accepting gambles with negative expected monetary returns is also prevalent in lotteries. The average returns from winning the jackpots of Saturday, Oz, and Powerball Lotto in financial year 1998/99 fall far short of what the objective odds would imply. The objective odds of winning the jackpots are about 8.15m to 1 in the Saturday and Oz Lotto, and about 55m to 1 in Powerball Lotto while the average winnings for each dollar invested were $0.57m, $2.69m, and $5.54m per dollar bet, respectively.

III. Risk Loving or Love of Skewness?

There are two explanations as to why rational individuals would accept gambles with negative expected returns. The first explanation, which can be considered as the orthodox explanation and which follows from the von Neumann-Morgenstern utility theory, suggests that gamblers accepting gambles with negative expected returns are risk-loving individuals (Quandt, 1986, Kanto et al., 1992, and Hamid et al., 1996). The second explanation, deriving from the seminal work of Friedman & Savage (1948), suggests that risk-averse individuals may indulge in unfair gambles if winning will significantly improve their standard of living. That is, risk-averse individuals are attracted to the skewness of the pay-offs.

According to economic theory, the most common observed patterns of behaviour of rational individuals are (a) they will not harm their own interests, (b) they are risk averse, and (c) they probably possess decreasing absolute risk aversion. The first feature seems to be apparent. It implies that no one in a right mind would participate in gambles or investments with negative expected returns. The second feature is also quite easy to explain: Almost everybody buys insurance (say, house and content insurance) to protect themselves from possible financial losses. Furthermore, a relatively rich individual would be willing to pay a lower insurance premium than that of a relatively poor individual when confronted with the similar down side of financial risk. The decrease in willingness to pay for insurance premium is known to economists as decreasing absolute risk aversion. In mathematical terms, if individuals possess von Neumann-Morgenstern utility functions $U(W)$ where W are the levels of initial wealth, then we expect $U'(W) > 0$, $U''(W) < 0$, and

$$\frac{d}{d}\left(\frac{-U''}{U'}\right)=\frac{-U'U'''+(U'')^2}{(U')^2}<0. \tag{2}$$

Note that $U'(W) > 0$, so equation (2) has a negative sign if and only if $U'''(W) > 0$, which implies that an risk-averse individual with decreasing absolute risk aversion must prefer positive skewness (see Arditti, 1967)[8]. Therefore, the two explanations advanced above must be incompatible with each other. Which of the two explanations is more plausible than the other?

The importance of Arditti's suggestion that $U'''(W) > 0$ lies in the third central moment of the utility function. To make the connection between $U'''(W)$ and the third central moment, one can approximate the utility function $U(W+X)$ with the gamble X by a Taylor series truncated to three terms. Suppose an individual has an initial wealth of W and is confronted by a gamble X with $E(X)\neq 0$ then the truncated Taylor series of $U(W+X)$ is

$$U(W+X)=\frac{U(W)}{0!}+\frac{U'(W)}{1!}X+\frac{U''(W)}{2!}X^2+\frac{U'''(W)}{3!}X^3 \tag{3}$$

Taking expectation, gives

$$EU(W+X)=U(W)+\frac{U'(W)}{1!}m_1+\frac{U''(W)}{2!}m_2+\frac{U'''(W)}{3!}m_3 \tag{4}$$

where m_1 is the mean, m_2 is the variance, and m_3 is the skewness[9]. Since the presence or absence of the gamble will not affect the initial utility $U(W) > 0$, it can be regarded as a constant. Now, holding variance m_2 constant, a decrease in the expected returns of the gamble, m_1, can be compensated by an increase of the skewness, m_3, to maintain the expected utility $EU(W + X)$, and vice versa. This makes sense when we relate this finding to horseracing and lotteries. Equation (2) and (4) jointly imply that individuals will accept lower expected returns in exchange with a higher skewness of returns with variance constant.

When the pay-off distribution is so skewed to the right (i.e., the right tail of the distribution is much longer than the left tail), expected return may even become negative. This explains the phenomenon that is described in Section II.

Samuelson (1967) raises another question about using the mean-variance approach to study the phenomenon of long shot bias. He points out that as long as the risk involved is symmetric and is small relatively to wealth of the individuals concerned, the expected utility functions can be approximated by the first two central moments, that is, the third term on the RHS of equation (4) disappears. As the risk to wealth increases, the first two central moments would become less and less accurate in estimating the expected utility of the gamble and higher central moments have to be considered.

When individuals bet on a long shot (say, defined by a market odd of 18 to 1 up to 200 to 1), the potential pay-off is between \$18 to \$200 for a \$1 bet. When individuals purchase lotto tickets, they are expecting a potential pay-off of at least \$1m. Considering the entry prices to these gambles are often very small, the risk involved is highly asymmetric with a relatively small down side risk but an extremely high up side risk. The behaviour of these individuals is consistent with a mean-variance framework of utility maximisation if and only if bettors are risk lovers. With the third term on the right-hand-side of equation (4) omitted as by Quandt (1986) and Kanto et al. (1992) in their estimation of the expected utilities of long shots, inaccuracies creep in. As a result, a decrease of expected returns, m_1, given the value of the variance, m_2, can only be compensated by an increase in $U''(W)$, which may have contributed to their finding of risk-loving bettors. Therefore the expected utility of betting on long shots or buying a lotto ticket can not be accurately estimated by the mean-variance approach to expected utility. It has to be extended beyond the second central moment to include the third central moment.[10]

Insights from economic and finance literature provide some strong arguments for supporting the thesis that individuals accepting gambles with negative expected returns are attracted by the skewness of the pay-off rather than being risk loving. The next section examines some recent empirical evidence that also lends its support to the skewness argument.

IV. Recent Empirical Evidence

Recent empirical research on a mean-variance-skewness approach to individuals facing risk confirms that skewness is statistically significant in the regression equation.

Purfield & Waldron (1997) use the two stage least square technique to study the impact of skewness on the demand for 577 Irish lotto draws from May 1990 to December 1995. It is useful to consider that the demand for lotto tickets is determined by the expected utility of the pay-offs of the winning tickets while the entry price is the expected utility associated with the losing tickets. To ensure that the influence of the skewness of the pay-offs of the lotto jackpot is properly accounted for, a list of non-monetary factors is also examined.[11] The estimated coefficients for the mean, variance, and skewness are of the correct signs and are all statistically significant.

Golec & Tamarkin (1998) use data from several U.S. racecourses for the years 1990-92 to examine the reason for the existence of long shot bias. Their data showed that when the market odds of the winner lengthened, it was almost certainly (with only a few exceptions) accompanied by an increase in variance and skewness. And it was skewness that pulled up the expected value of the winning tickets. These authors show that the empirical results of earlier studies on long shot bias (Ali, 1977; Hausch et al., 1981; Asch & Quandt, 1987) all show a consistent pattern of variance and skewness increasing (skewness increases faster than variance) as market probabilities decline. They conclude that the phenomenon of long shot bias may be explained by bettors' preference for return skewness.

Garrett & Sobel (1999) extend the work of Golec & Tamarkin (1998) to lotteries by examining the significance of return skewness to the expected utility of the lottery players. They use data from 216 on-line lottery games offered in the U.S. during 1995 for their empirical test. They divide the 216 lottery games into two sub-samples, one with 112 games offering top prizes of less than $10,000, and one with 104 games offering top prizes of more than $10,000. This allows for the investigation of whether risk and skewness preferences differ if one considers different magnitudes of top prize payouts. The regression results show statistically significant coefficients for all three moments for each of the three samples. In addition, the estimated values for expected returns (skewness) are the lowest (highest) in the sub-sample where the top prizes exceed $10,000. These results confirm that there is a trade-off between the expected returns and skewness.

V. Optimal Design of Gambling Games

This section discusses the implication of the trade-off between expected returns and skewness on the optimal design of gambles. Since risk-averse individuals are attracted to positively and highly skewed pay-offs, the designers of gambling games need to heed to this characteristic. Quiggin (1991, p.8) suggests that for a single-prize lottery[12], "the optimal solution would be an infinitely large prize with an infinitesimally small chance of winning". This skewness principle is also applicable to gambling games with a number of prizes. As long as a very large top prize (with very long odds of winning) dominates the prize structure, the set of small prizes does not make much difference to the expected value of the gamble. Therefore, individuals can focus on the top prize provided the top prize is much greater than any one of the small prizes. As a matter of fact, the evolution of the design of new gambling games seems to follow closely to this skewness principle.

An examination of the evolution of the format of Australian Lotto Bloc lotto games reveals the odds of winning the top prize in Powerball Lotto (introduced in 1996) lengthened from about 8.15m to 1 in the regular lotto game (Saturday and Oz Lotto) to about 55m. to 1. This is mainly achieved by having the powerball drawn from a separate barrel. As a result, there are fewer winners and a lot more rollovers, and the monetary value of the jackpot of Powerball Lotto increases substantially from that of the regular lotto games. A similar pattern of evolution arises in the horseracing in Hong Kong. Exotic bets with various odds of winning the top prizes have been introduced during the last two decades. These games can be seen as representing a strategy of product differentiation, trying to capture all consumers with various preferences by the skewness of prize structures.

Our observation of the evolution of the prize structure of the gambling games suggests that new gambling games are designed in such a way (more and more positively skewed prize structures with longer and longer odds to win the top prizes) to capture the imagination of the gamblers. However, designers of gambling games should pay attention to the trade-off between the perception of ease of winning and the size of the major prize. Take Powerball Lotto as an example, it has the highest average Division 1 prize[13], yet it is not the most popular lotto game in Australia because there are so few winners. For the sample we have from Draw 33 to Draw 141, there were only 29 draws that produced a winner, that is, only one out of every four draws. Larger and larger major prize can only be created by more and more positively skewed prize structure with longer and longer odds to win the major prize. But when the odds of winning the major prize is lengthened too much and there is a lack of retrievability of instances (not enough winners), gamblers' enthusiasm of participating in the game fades. There seems to be an optimal trade-off between revenue and skewness of the pay-off. If the prize structure is too skewed and there are not enough winners, the game will not be played by as many participants. However, if the major prize is not perceived as big enough, the game will not attract gamblers, therefore, lowering sales.

VI. Conclusion

This paper evaluates two explanations to the seemingly irrational behaviour of individuals attracted to gambling games with negative expected returns. Both from a theoretical and empirical point of view, it is found that the skewness explanation is far superior to the mean-variance explanation.

The present paper also examines the evolution of the design of gambling games (especially in horseracing and lotteries). It is observed that the evolution seems to follow closely the skewness principle: the optimal design includes an infinitely large prize with infinitesimally long odds of winning. However, it is also observed that there is a trade-off between sales and the skewness of the major prize; the lack of retrievability of instances (not enough winners) turns individuals away from participating in the game.

Notes

1. Double requires the bettors to correctly select the first place horses of two nominated races (or legs).

2. Six Up requires the bettors to correctly select either the first or second place horses in each of the six legs.

3. Double Quinella requires the bettors to correctly select the first and second place horses in either order in both legs.

4. Treble requires the bettors to correctly select the first place horses in three nominated races.

5. Double Trio requires the bettors to correctly select the first, second, and third place horses in any order in both legs.

6. Triple Trio requires the bettors to correctly select the first, second, and third place horses in any order in three nominated races.

7. The odds of the i^{th} horse winning in a race is $O_i = \frac{1-t}{p_i} - 1$, where t is percentage of the operator take and government taxation, p_i is the market probability of winning, which can be calculated by finding the ratio of the bet on the i^{th} horse over the total betting pool of the race.

8. Arditti (1967) deals with required returns for investment with risk. Interesting, the study of gambling and financial investment behaviour share similar methodology. Economists have long realised that the study of gambling behaviour lends much support to the study of financial markets, because like financial investments gambling involves the transfer of money among parties based on the outcome of some contingencies. Gambling markets are better suited to study individuals' risk preferences because each bet has a well-defined termination point at which its value becomes certain. The absence of this property in financial markets has made it difficult to test for market efficiency and rationality in the financial markets particularly in the stock market.

9. A unit free measure of skewness, a_3, is the quotient of the third central moment, m_3 (which is not unit free), divided by the cube of the standard deviation, denoted by s^3 or $m_2^{3/2}$. That is, $a_3 = \frac{m_3}{m_2^{3/2}} = \frac{E(W - E(W))^3}{m_2^{3/2}}$.

   For a symmetric distribution, we have $a_3 = 0$. A negatively skewed (or skewed to the left) distribution implies $a_3 < 0$ while a positively skewed (or skewed to the right) distribution, $a_3 > 0$. A distribution is highly skewed if $|a| > 1$.

10. There is no meaningful implication to include the fourth or higher central moment into the approximation of the expected utility.

11. The list of non-monetary variables included dummy variables account for different game formats, jackpot guarantees, number of rollovers, seasonal variation, bonus draws, etc.

12. In reality, there are gambling games that have a more even distribution of prizes catering for individuals that have a less distorted probability-weighting function (see Tversky & Kahneman, 1992). One can argue that in single-prize gambling games, the highly skewed returns musty be accompanied by very low objective probabilities to ensure that the expected values are lower than the entry prizes to allow for the operators' take and government taxes. To make this kind of gambling game worthwhile in participation, the gamblers' subjective probabilities must be distorted to produce positive expected utilities.

13. The largest jackpot ever won in Australia by one single (system 6) ticket is $30m. in Draw 245 on 25 January 2001.

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